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Thermally Asymmetric Triangular Fin Analysis

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The nondimensional heat loss from a geometrically symmetric, but thermally asymmetric, triangular fin is investigated as a function of the nondimensional fin length and a ratio of upper and lower surface Biot numbers (*Bi*2/*Bi*1) using the two-dimensional separation of variables method. The comparison between the nondimensional temperature for the thermally asymmetric condition and that for the thermally symmetric condition is made. The relationship between the nondimensional fin length and the ratio *Bi*2/*Bi*1 for equal amounts of heat loss is also discussed, as well as the relationship between *Bi*2/*Bi*1 and *Bi*1 for equal amounts of heat loss.

Nomenclature

$Bi1$	=	fin upper surface Biot number, $h_1 l/k$
$Bi2$	=	fin bottom surface Biot number, $h_2 l/k$
h_1	=	fin upper surface convection coefficient, W/m$^{2\circ}$C
h_2	=	fin bottom surface convection coefficient, W/m$^{2\circ}$C
k	=	thermal conductivity, W/m°C
L	=	nondimensional fin size, L'/l
L'	=	fin length, m
l	=	one-half fin root height, m
Q	=	heat loss, W
T	=	temperature, °C
T_w	=	fin root temperature, °C
T_∞	=	ambient temperature, °C
x	=	nondimensional coordinate along the fin length, x'/l
x'	=	coordinate along the fin length, m
y	=	nondimensional coordinate along the fin height, y'/l
y'	=	coordinate along the fin height, m
θ	=	nondimensional temperature $(T - T_\infty)/(T_w - T_\infty)$
θ_0	=	adjusted fin root temperature $(T_w - T_\infty)$, °C
λ_n	=	eigenvalues where $n = 1, 2, 3, \ldots$

Introduction

FINNED surfaces are widely used to enhance the rate of heat transfer to a surrounding fluid in many applications such as the cooling of combustion engines, electronic equipment, heat exchangers, and so on. As a result, a great deal of attention has been directed to fin problems. Various shapes of fins have been studied. For example, Chung and Iyer,[1] Look and Kang,[2] and Look[3] have discussed rectangular fins; Aziz and Nguyen,[4] Burmeister,[5] and Abrate and Newnham[6] were concerned with triangular fins; and Kang and Look[7] and Kraus et al.[8] examined trapezoidal fins. Finally, Ullmann and Kalman[9] and Lau and Tan[10] researched annular fins. Usually, most of the studies on the fin assume that the heat transfer coefficients for all surfaces of the fin are the same. However, Look and Kang[2] dealt with thermally asymmetric rectangular fins. However, no literature seems to be available that presents a triangular fin with unequal heat transfer coefficients.

This paper presents an analysis of a thermally asymmetric triangular fin. In this study, the upper surface Biot number $Bi1$ is equal to or larger than the bottom surface Biot number $Bi2$ and, as a mathematic convenience, the average value is taken at the fin tip. The nondimensional heat losses are investigated as a function of the nondimensional fin length and the $Bi2/Bi1$ ratio using the two-dimensional separation of variables method. The comparison between the nondimensional temperature profile for thermally asymmetric conditions and that for thermally symmetric conditions is made. Further, for arbitrary values of the nondimensional fin length and Biot numbers, the relation between the nondimensional fin length and $Bi2/Bi1$ for equal amounts of heat loss is shown. Further, the relationship between $Bi2/Bi1$ and $Bi1$ for equal amounts of heat loss is also presented. For simplicity, steady state as well as constant root temperature and thermal conductivity of the fin's material are assumed.

Two-Dimensional Analysis

For the geometrically symmetric, but thermally asymmetric, triangular fin shown in Fig. 1, the governing differential equation is

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0 \tag{1}$$

Two standard boundary conditions and two energy balance equations complete the problem formulation. They are

$$\theta = 1 \qquad \text{at} \qquad x = 0 \tag{2}$$

$$\frac{\partial\theta}{\partial x} + \frac{1}{2}(Bi1 + Bi2)\cdot\theta = 0 \qquad \text{at} \qquad x = L \tag{3}$$

$$-\int_0^1 \left.\frac{\partial\theta}{\partial x}\right|_{x=0} \mathrm{d}y - \int_0^L \left.\frac{\partial\theta}{\partial y}\right|_{y=0} \mathrm{d}x = Bi1\cdot\sqrt{1+L^2}\int_0^1 \theta\,\mathrm{d}y \tag{4}$$

$$-\int_{-1}^1 \left.\frac{\partial\theta}{\partial x}\right|_{x=0} \mathrm{d}y = Bi1\cdot\sqrt{1+L^2}\int_0^1 \theta\,\mathrm{d}y + Bi2\cdot\sqrt{1+L^2}\int_{-1}^0 \theta\,\mathrm{d}y \tag{5}$$

Equation (2) is the standard wall condition, whereas Eq. (3) is a condition at the tip required to complete the formulation. For this steady-state application, Eq. (4) is an application of the conservation of energy in the upper-half of the fin, whereas Eq. (5) is the required steady-state requirement of the energy conducted into the fin being convected away from the slanted surfaces. The applications of Eqs. (4) and (5) are true on the average. Furthermore, they are an

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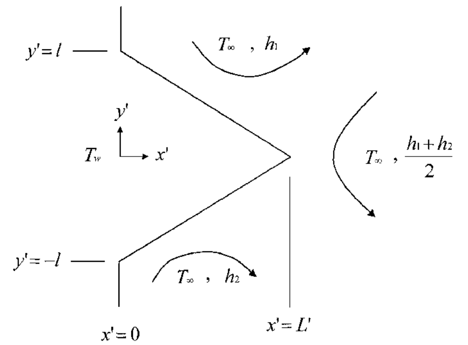

Fig. 1 Geometry of a thermally asymmetric triangular fin.

application of the first law of thermodynamics, which provides a more convenient condition than an energy balance on a differential control volume at the upper or lower surface of the fin.

When Eq. (1) with two boundary conditions [Eqs. (2) and (3)] and the first energy balance equation (4) are solved, the nondimensional temperature can be obtained by the separation of variables procedure. The result is

$$\theta(x, y) = \sum_{n=1}^{\infty} N_n \cdot f(x) \cdot f(y) \tag{6}$$

where

$$f(x) = \cosh(\lambda_n x) - f_n \cdot \sinh(\lambda_n x) \tag{7}$$

$$f(y) = \cos(\lambda_n y) + g_n \cdot \sin(\lambda_n y) \tag{8}$$

$$N_n = \frac{4\sin(\lambda_n)}{\left\{[2\lambda_n + \sin(2\lambda_n)] + g_n^2 \cdot [2\lambda_n - \sin(2\lambda_n)]\right\}} \tag{9}$$

and f_n and g_n are expressed by

$$f_n = \frac{\lambda_n \cdot \tanh(\lambda_n L) + \frac{1}{2}(Bi1 + Bi2)}{\lambda_n + \frac{1}{2}(Bi1 + Bi2) \cdot \tanh(\lambda_n L)} \tag{10}$$

$$g_n = \frac{2\lambda_n \cdot Bi1 \cdot AA_n + Bi1 \cdot (Bi1 + Bi2) \cdot BB_n - \lambda_n \cdot \sqrt{1 + L^2} \cdot CC_n}{2\lambda_n \cdot Bi1 \cdot DD_n + Bi1 \cdot (Bi1 + Bi2) \cdot EE_n + \lambda_n \cdot \sqrt{1 + L^2} \cdot FF_n} \tag{11}$$

AA_n–FF_n shown in Eq. (11) are

$$AA_n = L \cdot \cos(\lambda_n) \cdot \sinh(\lambda_n L) + \sin(\lambda_n) \cdot \cosh(\lambda_n L) \tag{12}$$

$$BB_n = L \cdot \cos(\lambda_n) \cdot \cosh(\lambda_n L) + \sin(\lambda_n) \cdot \sinh(\lambda_n L) - L \tag{13}$$

$$CC_n = 2\lambda_n \cdot \sin(\lambda_n) \cdot \sinh(\lambda_n L) + (Bi1 + Bi2) \cdot \sin(\lambda_n) \times \cosh(\lambda_n L) \tag{14}$$

$$DD_n = \cos(\lambda_n) \cdot \cosh(\lambda_n L) - L \cdot \sin(\lambda_n) \cdot \sinh(\lambda_n L) - 1 \tag{15}$$

$$EE_n = \cos(\lambda_n) \cdot \sinh(\lambda_n L) - L \cdot \sin(\lambda_n) \cdot \cosh(\lambda_n L) \tag{16}$$

$$FF_n = (Bi1 + Bi2) \cdot \{1 - \cos(\lambda_n) \cdot \cosh(\lambda_n L)\} - 2\lambda_n \cdot \cos(\lambda_n) \times \sinh(\lambda_n L) \tag{17}$$

Eigenvalues λ_n are obtained by using the last energy balance equation (5). The result of this manipulation is

$$2f_n \cdot \lambda_n \cdot \sin(\lambda_n) \cdot \sqrt{1 + L^2} = (GG_n - f_n \cdot L \cdot HH_n) \cdot (Bi1 + Bi2) + g_n \cdot (HH_n - f_n \cdot II_n) \cdot (Bi1 - Bi2) \tag{18}$$

where

$$GG_n = \sin(\lambda_n) + L \cdot \sinh(\lambda_n L) \tag{19}$$

$$HH_n = \cosh(\lambda_n L) - \cos(\lambda_n) \tag{20}$$

$$II_n = \sinh(\lambda_n L) - L \cdot \sin(\lambda_n) \tag{21}$$

Finally, the value of the heat loss from the thermally asymmetric triangular fin can be calculated by the application of Fourier's law. This result is

$$Q = 2k\theta_0 \sum_{n=1}^{\infty} N_n \cdot f_n \cdot \sin(\lambda_n) \tag{22}$$

Results

The variation of the nondimensional heat loss from a thermally asymmetric triangular fin is presented in Fig. 2 as the ratio $Bi2/Bi1$ varies from 0.9 to 1.0 in the case of $Bi1 = 0.01$, 0.1, and 0.2 for a typical value of $L = 5$ (note that when $Bi2/Bi1$ equals 1, the triangular fin becomes thermally symmetric). The nondimensional heat loss increases linearly as $Bi2/Bi1$ increases for all three values of $Bi1$.

The variation of the nondimensional heat loss from a thermally asymmetric triangular fin is shown in Fig. 3 as the nondimensional fin size varies from 1 to 10 in the case of $Bi2 = 0.001$, 0.005, and 0.01 for $Bi1 = 0.01$. Physically, $Bi2 = 0.005$ means that the heat transfer coefficient of the bottom surface is half of that of the top surface, whereas the fin is thermally symmetric for $Bi2 = 0.01$. When the heat loss for $Bi2 = 0.01$ and 0.005 is compared, the heat loss ratio increases as the nondimensional fin size varies from 1 to 10. The ratio of heat loss for $Bi2 = 0.001$ to that for $Bi2 = 0.01$ also increases for this range.

Figure 4 may be used to compare the variation of the nondimensional temperature along the upper and lower surface with the centerline profile for $L = 5$, $Bi1 = 0.11$ and $Bi2 = 0.09$. Note that the temperature along the upper fin surface is lowest, whereas along the centerline, the profile is the highest; the differences of these values decrease as the x coordinate increases. The ratio of the nondimensional temperature, as presented in Fig. 4, to that for the thermally symmetric temperature case, that is $Bi1 = Bi2 = 0.1$,

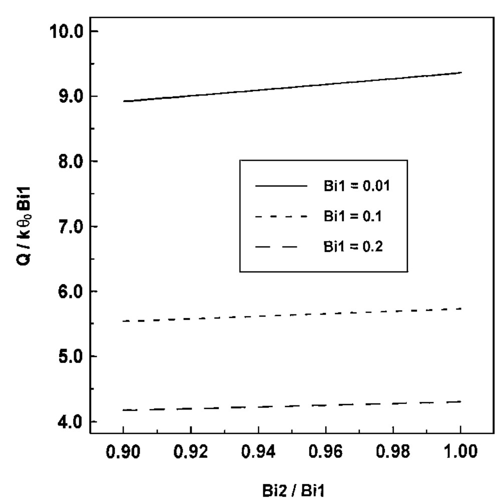

Fig. 2 Nondimensional heat loss vs *Bi2*/*Bi1* for a typical fin size ratio of *L* = 5.

Table 1 Ratio of the nondimensional thermally asymmetric temperature to the symmetric case temperature for L = 5 at various positions in the fin[a]

x	Upper surface y, %	Centerline y, %	Lower surface y, %
0.1	99.02 (0.98)	100 (0)	100.98 (−0.98)
1	99.20 (0.8)	100.01 (0)	100.82 (−0.8)
2	99.42 (0.6)	100.02 (0)	100.62 (−0.6)
3	99.63 (0.4)	100.03 (0)	100.43 (−0.4)
4	99.84 (0.2)	100.03 (0)	100.23 (−0.2)
5	100.04 (0)	100.04 (0)	100.04 (0)

[a](Θ for $Bi = 0.11$ and $Bi2 = 0.09$) / (Θ for $Bi1 = Bi2 = 0.1$).

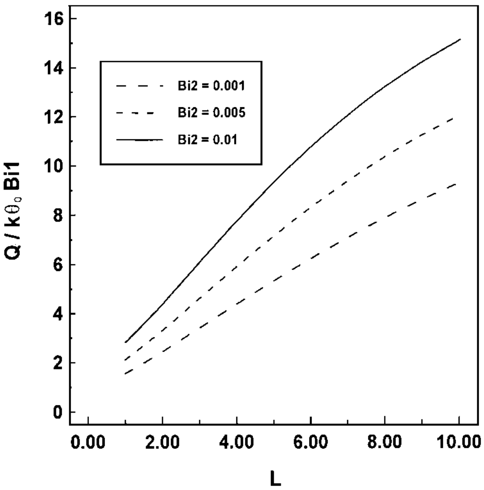

Fig. 3 Nondimensional heat loss vs L for $Bi1$ = 0.01.

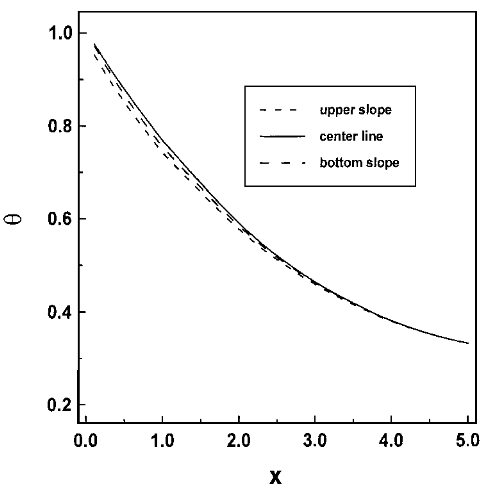

Fig. 4 Temperature variation along the x coordinate for L = 5, $Bi1$ = 0.11, and $Bi2$ = 0.09.

is listed in Table 1. The ratio along the upper surface varies from 99.02 to 99.84%, whereas the ratio along the lower surface varies from 100.98 to 100.23%; along the centerline this ratio is essentially 100% as x increases from 0.1 to 5. Table 1 also shows that the nondimensional temperature, for thermally asymmetric conditions, is slightly larger than that for thermally symmetric condition case at the fin tip when the average Biot number, that is, $(Bi1 + Bi2)/2$, is equal to 0.1.

The relationship between the nondimensional fin size L and the Biot number ratio $Bi2/Bi1$ for equal amounts of heat loss is presented in Fig. 5. For $Bi1 = 0.005$ and $Bi2/Bi1 = 1$, approximately 2.21 times the nondimensional fin length for $Bi1 = Bi2 = 0.01$ is needed to produce an equal amount of heat loss. It also indicates that the nondimensional fin size is approximately $\frac{1}{3.8}$ that for $Bi1 = Bi2 = 0.005$ when $Bi1 = 0.015$ and $Bi2/Bi1 = 1.0$.

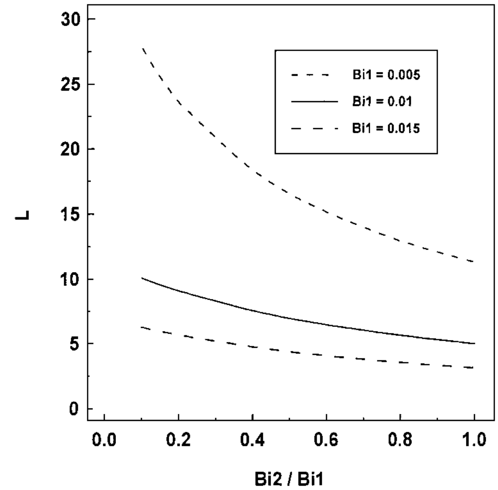

Fig. 5 Relation between L and $Bi2/Bi1$ for equal amounts of heat loss.

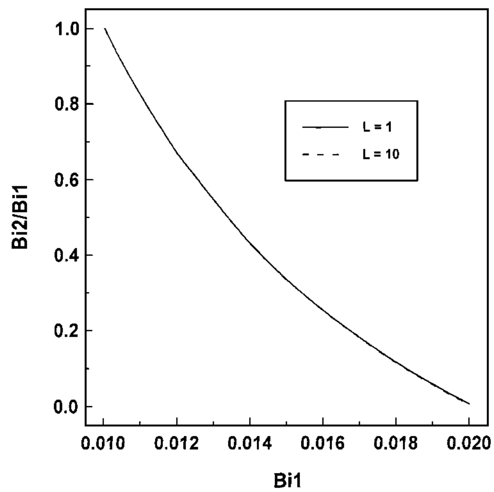

Fig. 6 Relation between $Bi2/Bi1$ and $Bi1$ for equal amounts of heat loss in case of $Bi1$ = $Bi2$ = 0.01.

Finally, the relationship between $Bi2/Bi1$ and $Bi1$ to produce the equal amounts of heat loss is shown in Fig. 6 for $Bi1 = Bi2 = 0.01$ for L equals, 1 and 10. Note that this relationship seems to be independent of the nondimensional fin length.

Conclusions

From the two-dimensional analysis of symmetric, but thermally asymmetric, triangular fins presented here, the following conclusions can be drawn:

1) The nondimensional heat loss increases linearly as $Bi2/Bi1$ increases.

2) The nondimensional fin tip temperature for the thermally asymmetric case is slightly higher than that for thermally symmetric case.

3) The relation between $Bi2/Bi1$ and $Bi1$ for equal amounts of heat loss seems to be independent on the nondimensional fin length.

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